**1.**

The exact price of geometric Asian call option is

>> AsianGeomCall

price =

**4.0655**

**2.**

|  |  |  |
| --- | --- | --- |
|  | Crude MC | Control MC |
| 4.5564 | 4.3522 |
| 4.1762 | 4.3374 |
| 4.3601 | 4.3416 |
| 4.2992 | 4.3493 |
| 4.1039 | 4.3452 |
| 4.3855 | 4.3376 |
| 4.0470 | 4.3398 |
| 4.0604 | 4.3360 |
| 4.3605 | 4.3496 |
| 4.1892 | 4.3351 |
| mean | 4.2538 | 4.3424 |
| std | 0.1655 | 0.0063 |

95% confidence interval is [mean ±1.96\*std/sqrt(n)]

So 95% confidence interval for Crude **Monte Carlo is [4.1513, 4.3564]**;

**For Control Monte Carlo is [4.3385, 4.3463]**

|  |  |  |
| --- | --- | --- |
|  | Crude MC | Control MC |
| MSE | 0.02465 | 0.00004 |

Comparing Crude Monte Carlo Control Monte Carlo has a less standard deviation and a less mean square error. So Control Monte Carlo method is better and improve the mean square error from 0.025 to 4e-5.

**Appendix:**

**Code for part 1,**

function AsianGeomCall

r=0.05;

S0=50;

K=50;

sigma=0.3;

n=10;

t=[0.1:0.1:1];

i=[1:1:10];

expSG=S0\*exp((r-sigma^2/2)\*(sum(t)/n)+sigma^2/2/n^2\*sum((2\*n+1).\*t-2.\*i.\*t));

var=sigma^2/n^2\*sum((2\*n+1).\*t-2\*i.\*t);

d1=(log(expSG/K)+var/2)/sqrt(var);

d2=(log(expSG/K)-var/2)/sqrt(var);

price=exp(-r\*1)\*(expSG\*normcdf(d1)-K\*normcdf(d2));

price

**Code for Asian arithmetic call option using Crude Monte Carlo**

function CrudeAsianAriCall

path = 1000;

num\_est = 10;

dim = 10;

x=zeros(dim,path);

option = zeros(1, num\_est);

for k = 1: num\_est

u=rand(dim,path);

for i = 1:path

for j = 1:2:dim

tempp = BoxMuller(u(j,i),u(j+1,i));

x(j,i)=tempp(1);

x(j+1,i)=tempp(2);

end

end

std = 0.3;

sumtemp = 0.0;

ssum=0.0;

s0 = 50;

r=0.05;

strike = 50;

for i = 1:path

s = s0;

for j = 1:dim

s = s \* exp( ( r - std \* std / 2 ) \* 0.1 + std \* sqrt(0.1) \* x( j , i ) );

ssum=ssum+s;

end

ssum=(ssum-50)/dim;

sumtemp = sumtemp + max( ssum - strike,0 );

end

option(1,k) = exp(-r)\*(sumtemp/path);

end

option'

end

**Code for Asian arithmetic call option using Control Monte Carlo**

function ControlAsianAriCall

dbstop if error

path = 1000;

num\_est = 10;

dim = 10;

x=zeros(dim,path);

C=4.0655;

option = zeros(1, num\_est);

for k = 1: num\_est

u=rand(dim,path);

for i = 1:path

for j = 1:2:dim

tempp = BoxMuller(u(j,i),u(j+1,i));

x(j,i)=tempp(1);

x(j+1,i)=tempp(2);

end

end

std = 0.3;

asum=0.0;

gsum=1.0;

s0 = 50;

r=0.05;

strike = 50;

a=[];

g=[];

for i = 1:path

s = s0;

for j = 1:dim

s = s \* exp( ( r - std \* std / 2 ) \* 0.1 + std \* sqrt(0.1) \* x( j , i ) );

asum=asum+s;

gsum=gsum\*s;

end

asum=(asum-50)/dim;

gsum=(gsum/50)^(1/dim);

a(i)= exp(-r)\*max( asum - strike,0 );

g(i)= exp(-r)\*max( gsum - strike,0 );

end

ag=(a-mean(a)).\*(g-mean(g));

gg=(g-mean(g)).\*(g-mean(g));

betatemp=sum(ag)/sum(gg);

option(1,k) = 1/path\*sum(a-betatemp.\*(g-C));

end

option'

end